An Econometric Method for Fitting Linear Integral Equations to Produce Short-Term Postal Forecasting Models

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Note: The views represented are solely those of the authors and not necessarily those of the Postal Regulatory Commission.

Short-run forecasting

- Traditional methods relate a future time path for a time series depicting sales or cost to a leading indicator that the series mimics with an expected lag.
- We recast the model as a pair of linear integral equations, where the variables and coefficients are represented by continuous functions defined over a specified interval of time.

- The equation takes the following general form:

$$y(u) = \alpha(u) + \sum_i \int_{s_i} x_i(v) A_i(u, v) dv + e(u) \quad for\ u \in R.$$

- The function $y(u)$ is assumed to be a real-valued function of a single variable, u, which is itself a real number within a region R which we equate to a continuous interval of time (a year in our example)
- There is no method for fitting a linear integral equation in the general form just described.
  - By transforming the equation to conform to the assumptions made by Pearsall (2022) we can apply matrix-algebraic-like methods to manipulate and solve particular linear integral equations.

Dependent Variable – Vol/CapSA.Mkt (y)

- Vol/CapSA.Mkt Marketing Mail APR Volume per Adult 16 years of Age and Older, Monthly Pieces, Seasonally Adjusted
- Derived from USPS' unaudited monthly financial reports filed with the PRC.
- To calculate "Vol/CapSA.Mkt," the aggregate volumes extracted from the reports are divided by the U.S. population aged 16 years and older, obtained from the U.S. Bureau of the Census.
- The series is seasonally adjusted using the X-12-ARIMA method developed by the U.S. Department of Commerce.

Independent Variables (1 of 3)

x_1 – VEC Error	Preceding Year Equation Error Function from Vector Error Correction Model co-integrating Equation

- Represents the difference over the prior year between the function for Vol/CapSA.Mkt and its prediction from the OLS (Ordinary Least Squares) fit of the co-integrating equation.
- Describes the deviation of Vol/CapSA.Mkt from its equilibrium level over the course of the previous year.

Independent Variables (2 of 3)

x_2 – UMCSENT(I)	University of Michigan: Consumer Sentiment, Index 1966: Q1=100, Monthly, Not Seasonally Adjusted, with Missing Values Interpolated
x_3 – PERMIT	Units Authorized in Permit-Issuing Places: Total Units, Thousands of Units, Monthly, Seasonally Adjusted Annual Rate New Privately-Owned Housing
x_4 – RAMTMNO	Real Manufacturers' New Orders: Total Manufacturing Matched, Millions of 2015 Dollars, Monthly, Seasonally Adjusted
x_5 – AWHMAN	Average Weekly Hours of Production and Nonsupervisory Employees, Manufacturing, Hours, Monthly, Seasonally Adjusted
x_6 – SPASTT01USM661N	Total Share Prices for All Shares for the United States, Index 2015=100, Monthly, Not Seasonally Adjusted

Independent Variables (3 of 3)

x_7 – FEDFUNDS	Federal Funds Effective Rate, Percent, Monthly, Not Seasonally Adjusted
x_8 – RealPrice.Mkt	Real Producer Price Index by Industry: U.S. Postal Service: Marketing Mail, Index Jun 1989=100, Monthly, Not Seasonally Adjusted
x_9 – ITUSERS	Internet Users for the United States, Per 100 People, Monthly Interpolated, Not Seasonally Adjusted

Adaptation of the Vector Error Correction (VEC) model

- The VEC model consists of two key components
  1. Co-integrating Equation:
     - Predictions from the co-integrating equation represent equilibrium levels that Vol/CapSA.Mkt approaches over time.
     - Provides insights into the long-term behavior of Vol/CapSA.Mkt.
  2. Forecasting Equation:
     - Describes single-year changes in Vol/CapSA.Mkt.
     - Includes a term relating the changes to the predicted disequilibrium of the previous period.
     - Captures short-term fluctuations and deviations from the long-term equilibrium.
- Our application converts all variables and coefficients of the conventional VEC model into functions.
- Projecting Vol/CapSA.Mkt for an entire year requires only a single evaluation of the estimated integral equations.

Overview of VEC Model Estimation Process

Subsidiary Estimations:

- Fit of integral equations using conventional econometric methods.
- Each equation in the set is an ordinary linear equation.

Data Adjustment:

- Elimination of non-economic effects from data.
- Effects removed: COVID-19 epidemic, election year months (September and October), 9-11 and anthrax attacks of 2001, class definition change in 2018, reporting period shift in 2002, and data anomaly in October 2018.

Preliminary Fits:

- Vector function $f(u)'=[1\ u\ u^2\ u^3\ u^4\ u^5\ u^6]$ used.
- Obtained using Weighted Least Squares (WLS).

Overview of VEC Model Estimation Process continued

VEC Model Fitting:

- Two-step OLS fit
  - Fit the co-integrating integral equation, and obtain estimates for the functions corresponding to VEC Error.
  - Fit the forecasting integral equation using the estimates from the previous step.

Autoregressive Process:

- AR-4 process selected, accounting for serial correlation in residuals.
- OLS fit of AR-4 process with significant coefficients.

Feasible Generalized Least Squares (GLS):

- Re-estimation of the forecasting equation using GLS, with AR-4 transformed sample.
- Calculate sample variance-covariance matrix from transformed residuals.
- Inverse of the matrix used for final GLS fits.

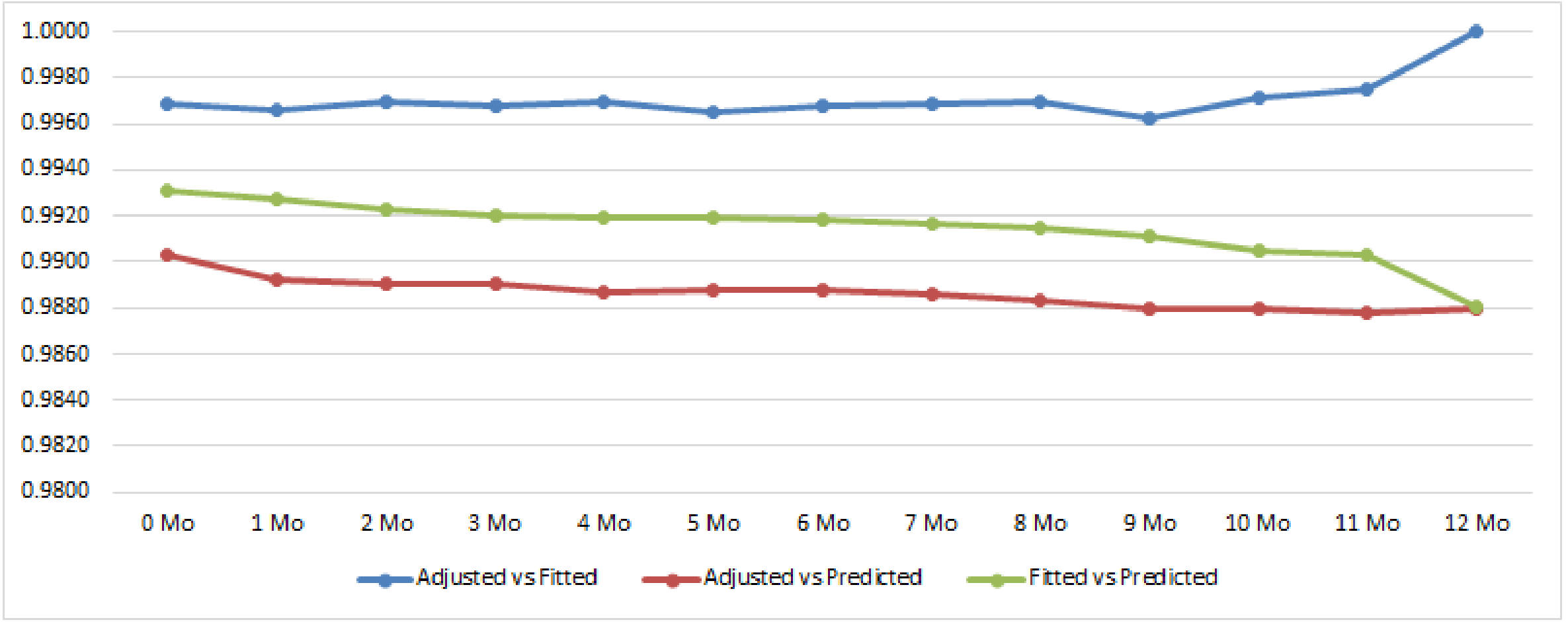
Statistical Dimensions of the Model and Fit

- OLS regressions used to fit the co-integrating equation use 602 observations to fit a linear equation with 57 variables including a constant term.
- Each of the forecasting equations contain 64 variables including a linear trend term.
- The VEC equations are fit to 590 observations .
- AR-4 equations are fit to 586 observations.
- Approximately one-fourth of the estimated coefficients and t-values are statistically significant.
- Statistically significant coefficients associated with every element of $f(u)$.
- Statistically significant coefficients in one or more of the equations for all seven of the components f0 through f6.

Goodness of Fit Statistics

	Co-Int.	VEC	AR-4		Co-Int.	VEC	AR-4
	Unudjusted	Unudjusted	Unudjusted		Adjusted	Adjusted	Adjusted
Component	R-Square	R-Square	R-Square		Std. Error	Std. Error	Std. Error
f0	0.9531	0.7236	0.6933		1.685	0.881	0.479
f1	0.4828	0.8233	0.7723		2.108	1.738	1.425
f2	0.2157	0.7878	0.7127		6.696	5.09	5.257
f3	0.2713	0.7832	0.7454		20.119	17.569	17.481
f4	0.1666	0.7141	0.6329		20.413	17.418	18.897
f5	0.2350	0.7497	0.6983		56.462	51.785	57.162
f6	0.2385	0.7411	0.6915		36.343	34.185	37.668

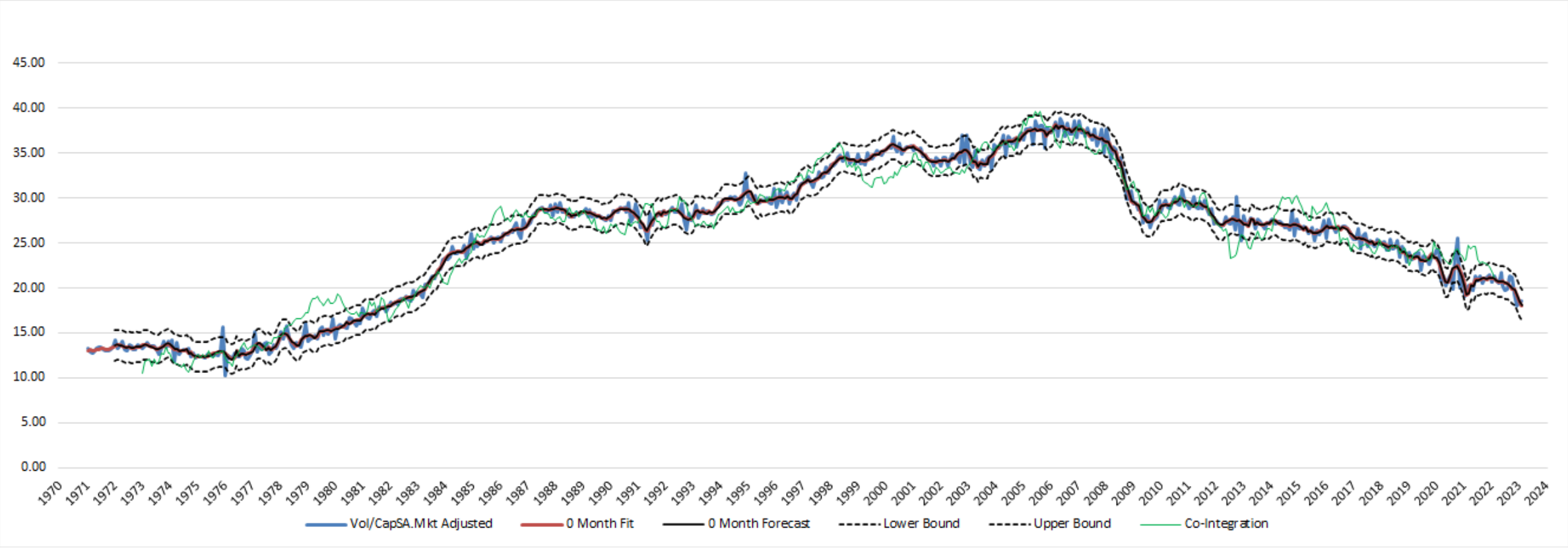
Correlations



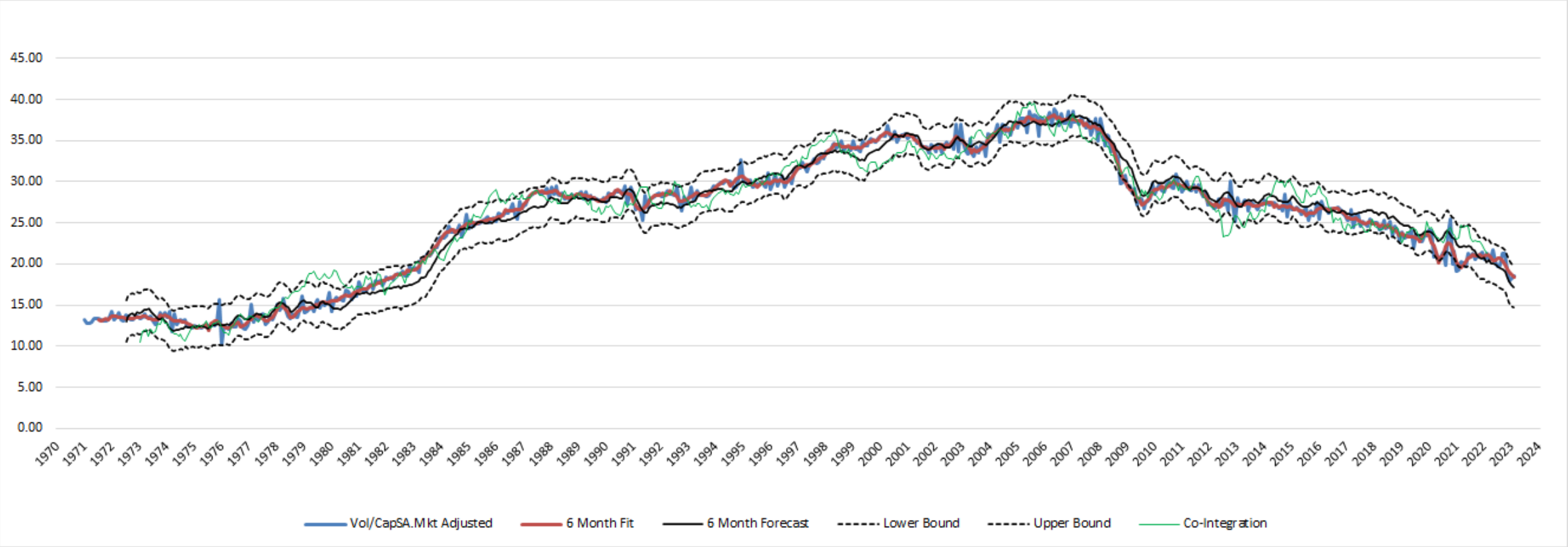
Forecasting Marketing Mail Volume per Capita

- For any data point in the sample, the AR-4 fit of the linear integral forecasting equation is used to generate a predicted function $y(u)$.
- $y(u)$ is a one-year time path for Vol/CapSA.Mkt beginning on the last date observed.
- $u \in [0,1]$ represents the proportion of the coming year corresponding to the forecast horizon.
- A forecast for any horizon may be produced simply by evaluating $y(u)$ using the value of u for the chosen horizon.

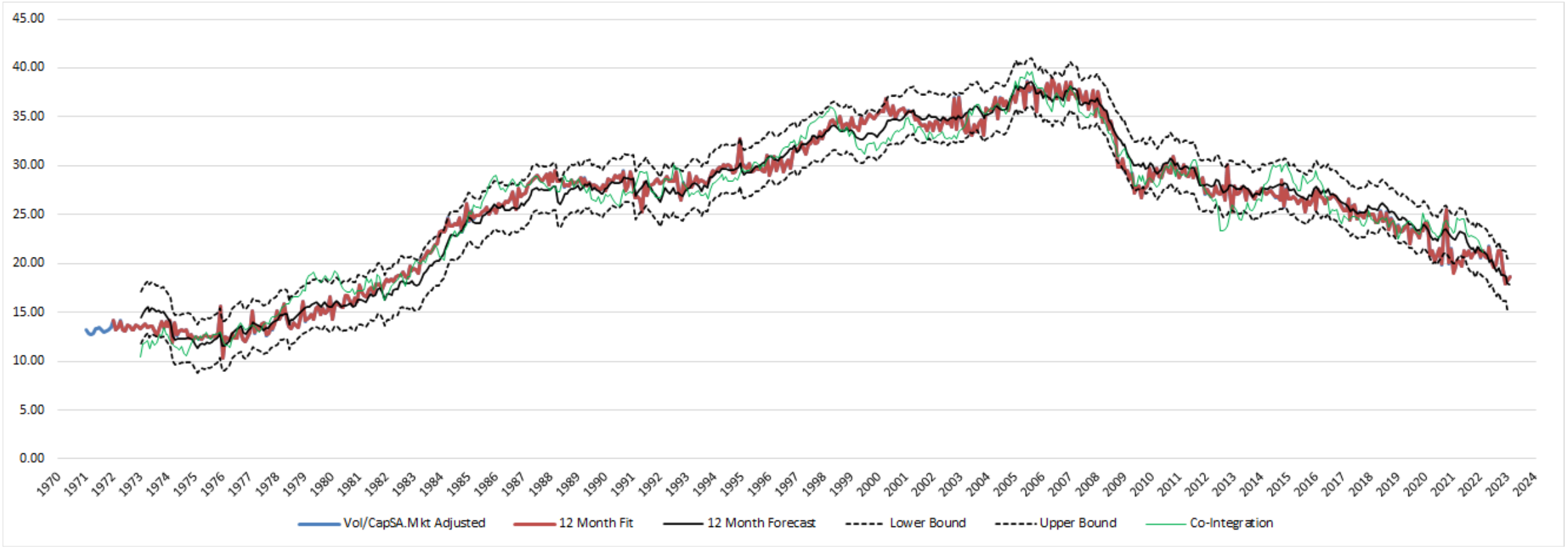
Zero-Month Forecasts of Vol/CapSA.Mkt (u=0)



Six-Month Forecasts of Vol/CapSA.Mkt



Twelve-Month Forecasts of Vol/CapSA.Mkt



February 1, 2023 to February 1, 2024 Forecasts of Vol/CapSA.Mkt